Skew Transpose

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April 16, 2005

1 Definition and Notation

The Skew Transpose is the transpose of a square matrix about the other, non-main diagonal (top right to bottom left). If A and B are $n \times n$ matrices, the skew transpose is defined as

$$\left(A^{\$}\right)_{ij} \equiv A_{(n+1-j)(n+1-i)}$$

2 Properties

2.1 $(AB)^{\$} = B^{\$}A^{\$}$

Proof

$$\begin{aligned} \left[(AB)^{\$}\right]_{ij} &= (AB)_{(n+1-j)(n+1-i)} \\ &= \sum_{k=1}^{n} A_{(n+1-j)k} B_{k(n+1-i)} \end{aligned}$$

Now, k is a dummy index that is summed over. As long as all the values it is summed over are hit, it can be expressed in any way that it convenient. The expression $n+1-k$ goes from n to 1 for k going from 1 to n, therefore, we can write

$$\left[(AB)^{\$}\right]_{ij} = \sum_{k=1}^{n} A_{(n+1-j)(n+1-k)} B_{(n+1-k)(n+1-i)}$$

$$
\begin{aligned}
&= \sum_{k=1}^{n} \left(A^{\$}\right)_{kj} \left(B^{\$}\right)_{ik} \\
&= \sum_{k=1}^{n} \left(B^{\$}\right)_{ik} \left(A^{\$}\right)_{kj} \\
&= \left(B^{\$} A^{\$}\right)_{ij}
\end{aligned}
$$