

Skew Transpose

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1 Definition and Notation

The Skew Transpose is the transpose of a square matrix about the other, non-main diagonal (top right to bottom left). If A and B are $n \times n$ matrices, the skew transpose is defined as

$$(A^\$)_{ij} \equiv A_{(n+1-j)(n+1-i)}$$

2 Properties

2.1 $(AB)^\$ = B^\$A^\$$

Proof

$$\begin{aligned} [(AB)^\$]_{ij} &= (AB)_{(n+1-j)(n+1-i)} \\ &= \sum_{k=1}^n A_{(n+1-j)k} B_{k(n+1-i)} \end{aligned}$$

Now, k is a dummy index that is summed over. As long as all the values it is summed over are hit, it can be expressed in any way that is convenient. The expression $n+1-k$ goes from n to 1 for k going from 1 to n , therefore, we can write

$$[(AB)^\$]_{ij} = \sum_{k=1}^n A_{(n+1-j)(n+1-k)} B_{(n+1-k)(n+1-i)}$$

$$\begin{aligned} &= \sum_{k=1}^n (A^{\$})_{kj} (B^{\$})_{ik} \\ &= \sum_{k=1}^n (B^{\$})_{ik} (A^{\$})_{kj} \\ &= (B^{\$}A^{\$})_{ij} \end{aligned}$$